

INVESTIGATION OF TEMPORAL VARIATIONS OF WINDS

Principal Investigators:

J. Bedinger

E. Constantinides

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FINAL REPORT
CONTRACT NO. NAS5-21036

Prepared for
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
GODDARD SPACE FLIGHT CENTER
GREENBELT, MARYLAND

August 1971

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FINAL REPORT
FOR
INVESTIGATION OF TEMPORAL VARIATIONS OF WINDS

Contract No. NAS5-21036

Goddard Space Flight Center

Contract Negotiator: Joseph J. Gentilini
Technical Officer: Wendell Smith

Prepared by:

GCA CORPORATION
GCA TECHNOLOGY DIVISION
Bedford, Massachusetts

Project Managers:

J. Bedinger
E. Constantinides

for

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INVESTIGATION OF TEMPORAL VARIATIONS OF WINDS

by J. Bedinger and E. Constantinides
GCA Corporation, GCA Technology Division,
Bedford, Massachusetts

ABSTRACT

This report discusses the continuing analysis of upper atmospheric wind data obtained from vapor trails. Data from a through-the-night series of seven vapor trails is analytical in terms of cyclic components for standard heights in order to describe the temporal variations of the winds. Evidence for the presence of components with 12 and 8-hour periods is presented. Similar analysis of all available data to date describes the average behavior of the winds throughout the night at Wallops Island.

A somewhat restricted model appropriate to the observation site (Wallops Island, Virginia) was constructed in order to investigate the physical properties of the atmosphere which are implied by the observed winds. The model utilizes the approximation of a plane-stratified atmosphere and allows derivation of formulas which express the variation of the density, pressure, temperature, and heating rates as a function of the observed winds. A perturbation method is used to estimate the magnitude of the non-linear, second order terms which result from the main diurnal mode between 80 and 100 km. Variation of 3 percent of the first order pressure value to 9 percent of the temperature were found.

It is concluded that the data from a small number of sequential observations (7 or less through the night) will allow an adequate description of the total wind variations only if the altitude and time variations are analyzed simultaneously. It is proposed that such methods be developed. It is further proposed that vapor trail measurements be closely coordinated with various other observations and that the method be extended for daytime usage.

SECTION I

INTRODUCTION

The objectives of this contract are: (1) to examine existing upper atmospheric wind data to determine the temporal variations of the winds, (2) to investigate the physical properties of the atmosphere implied from the above analysis and develop a particular model which describes the observed motions, and (3) to develop specific requirements for future observations and theoretical study.

The study of the temporal variation of the winds was initiated under Contract NAS5-11572. At that time, three series of vapor trails having five trails each and one series of six trails were available for analysis. The variation of the winds through the night were studied by utilizing the observed winds at specific intervals of time to obtain a continuous representation in terms of polynomials and harmonic components.

Also, a statistical analysis of the winds was obtained from a total of 57 vapor trails observed at Wallops Island. Forty-two of these trails occurred during twilight and fifteen were during the night. The results of both the temporal and statistical studies are described in the Final Report on Contract NAS5-11572 (GCA-TR-69-3-N).

Additional data for 15 more trails, eleven of them at night, have now been included in the analysis. Seven of these trails were spaced at about two-hour intervals covering a period of twelve hours during the night of 13-14 February 1969. The methods of analysis and results are discussed in Sections II and IV of this report.

Section III of this report discusses the preliminary results of the investigation of the physical properties of the atmosphere implied by the observed winds. The study is based on a model which is appropriate to the latitude of Wallops Island, Virginia and utilizes the approximation of a plane-stratified atmosphere. Formulas are obtained which express the variation of the density, pressure, temperature, and heating rate as a function of the observed winds. The model is also used to estimate the magnitude of the non-linear, second order terms which result from the main diurnal component between 80 and 100 km.

Section IV contains suggestions for the continued investigation of the dynamics of the thermosphere. Special emphasis is placed on a method of analysis which resulted from the study of the seven trail series and which is expected to greatly increase the information obtained from a series of sequential vapor trails. The method utilizes theoretical results and special smoothing procedures in order to analyze the altitude and time variation of the wind simultaneously. The coordination of vapor trail observations with other measurements and the extension of the method for daytime use are suggested.

SECTION II

TEMPORAL ANALYSIS OF WIND DATA

A. Introduction

This section contains the results of the temporal analysis of the observed winds. Two sets of data were used in the analysis. In the first, the data from the night of 13-14 February 1969 were analyzed in order to reveal the tidal components present on that occasion. The second set of data consisted of all the wind measurements taken at Wallops Island to date. In both cases, the original data from each trail were processed by a special computer program which introduced a minimal amount of smoothing and, in addition, tabulated values of the wind components at intervals of 0.5 km below 110 km and 1.0 km above 110 km. The temporal behavior of the winds was examined independently at each height, and for a fixed height, independently for each of the two horizontal components of the wind.

B. Data of 13-14 February 1969

On the night of 13-14 February 1969, seven wind measurements were carried out over Wallops Island, Virginia. The NASA rocket identification numbers, launch times, and time separations between successive launchings are listed in Table 1.

Table 1

LAUNCH TIMES AND TRAIL SEPARATIONS

| | | | | | | | |
|--------------------|-------|--------|--------|--------|--------|--------|--------|
| NASA Rocket Number | 18.86 | 14.397 | 14.399 | 14.400 | 14.401 | 14.398 | 14.402 |
| Launch Time (FST) | 18:11 | 20:00 | 22:19 | 00:00 | 02:00 | 04:00 | 06:13 |
| Time Separation | 01:49 | 02:19 | 01:41 | 02:00 | 02:00 | 02:13 | |

All of the trails covered the height range 92.5 to 152 km. The data from the seven trails in this height range were analyzed harmonically. Specifically, at each height, and separately for the eastward and northward wind components, the seven measurements were used to determine the parameters C_0 and C_n , δ_n , in the analytic representation:

$$F(t) = C_0 + \sum_{n=1}^N C_n \cos \omega_n (t - \delta_n)$$

In this equation $F(t)$ represents either the eastward or northward component, t is the time, $\omega_n = 2\pi/T_n$ is the frequency corresponding to the period T_n and the parameters C_n and δ_n represent, respectively, the amplitude and phase of the oscillation with period T_n . The parameter C_0 represents the prevailing wind. The method of analysis employed consists of determining the $(2N+1)$ parameters C_n ($n = 0$ to N) and δ_n ($n = 1$ to N) from the observations for a specified set of periods T_n ($n = 1$ to N). If the number of parameters equals the number of observations, then these parameters are determined uniquely, and the analytic representation reproduces the observed values exactly at the times of observation. If, however, the number of observations exceeds the number of free parameters, the latter are determined in a least-square sense, and the analytic representation of the data is approximate. In the latter case, the errors involved in the least-squares approximation provide an estimate of the appropriateness of the representation. In the former case no such estimate can be obtained.

The time series of 13-14 February 1969 consisted of seven observations. Hence, at most three periodic components, in addition to the prevailing component, can be used to represent the data from this series. The harmonic analysis of this series has been carried out using five distinct sets of three periodic components, three distinct sets of two periodic components, and two sets with only one periodic component. The following combinations of periods were used:

(12,8,6); (8,6,4); (12,6,4); (24,12,8); (16,9,7) for calculations involving three periodic components,

(12,8); (8,6); (9,7) for calculations involving two periodic components,

(12); (8) for calculations involving one periodic component.

In each case the numbers in parentheses give the period in hours.

The best combination was selected on the basis of satisfying the following two criteria: (1) The amplitudes C_n must have reasonable values (not exceeding 100 meters/sec), and (2) The value of the prevailing component, C_0 , must have roughly the same value as the average value of the seven observations. These criteria apply to the representation in terms of three periodic components. For the cases where less than three such components were used, the representation is an approximate one only. Hence, a third criterion is the requirement that the root-mean-square value of the residuals be reasonably small (not exceeding 15 meters/sec).

For the February 1969 series, for which the observations spanned 12 hours, the data cannot be represented adequately by a single periodic component. Representation in terms of two periodic components is mar-

ginally adequate if the two periods chosen are 12 and 8 hours. Among the representations involving three periodic components, the set using the periods 12, 8, and 6 hours is decidedly the best in the sense that it satisfies the two criteria mentioned above, whereas the other sets used fell far short of satisfying either of the two criteria. Further, the amplitudes and phases for the 12 and 8 hour components calculated using the three-component set of (12,8,6) is in reasonably good agreement with the amplitudes and phases calculated using the two-component set of (12,8), over most of the height range. This comparison may be an indication that the numerical results obtained from the calculation are, in fact, reasonably close to the actual amplitudes and phases of the 12 and 8 hour periodic components. These results are plotted in Figures 1 through 5.

Figure 1 is the altitude profile of the phases of the 12, 8, and 6-hour components of the eastward wind. (Phase is here defined as the hour at which the maximum value is achieved). Above 110 km the phase of the 12-hour component varies very smoothly. The rate of change of the phase corresponds to a vertical wavelength of 85 km. This value compares very well with the value of 88 km predicted by the tidal theory for the dominant semi-diurnal mode. Further, the phase decreases with height, implying a wave propagating with a downward phase velocity, which is associated with upward propagation of energy. These facts tend to indicate that the values derived from the analysis do in fact represent the actual semi-diurnal component. The phase of the 8-hour component also varies smoothly above 110 km. The rate of variation, although not as regular as in the case of the 12-hour component, implies a vertical wavelength of 80 ± 15 km. The first two modes of the 8-hour component have wavelengths of 125 km and 75 km, respectively, in this height interval. Hence, it is possible that the observed phase profile reflects the presence of one or both of these 8-hour modes. The present analysis cannot distinguish between these two possibilities. The phase of the 6-hour component is much less regular, and no unambiguous value of its wavelength can be derived from the data. In fact, this phase appears to undergo abrupt changes at 146 and 115 km, while varying only slightly in-between. Figure 2 shows that the amplitude of the 6-hour component has nodes at these heights. The combination of nodes and abrupt phase changes is associated with standing oscillations. These have been discussed in detail in Final Report on Contract NAS5-11572.

Below 110 km the phases of all three components vary less smoothly and tend, on the average, to increase with height. This would tend to indicate a downward propagation of energy in this region. If this conclusion is correct, then it would appear that energy was flowing away from the region around 110 km in both vertical directions, which would account for the relatively small eastward winds observed in this region throughout the night.

Figure 2 is the altitude profile of the amplitudes of the 12, 8, and 6-hour components of the eastward wind. Notable features are the fairly smooth variation of the 12 and 8-hour amplitudes above 110 km, the nodes of the 6-hour amplitude at 115 km and 146 km, and the relatively low amplitudes of all three components at 109-110 km. Below 110 km, no clear conclusions can be drawn from the behavior of the amplitudes.

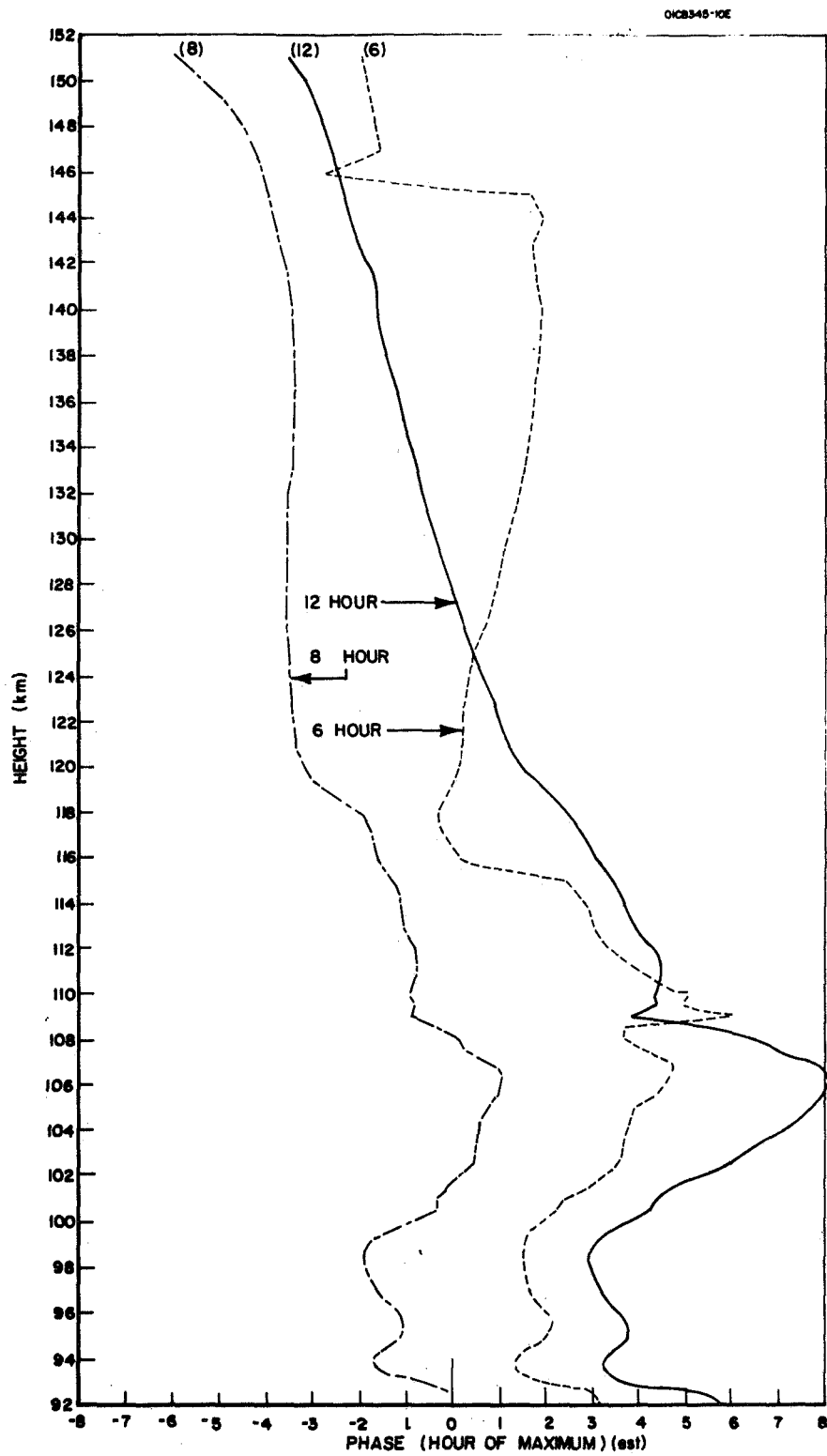


Figure 1. Phases of the 12, 8, and 6-hour components of the eastward wind.

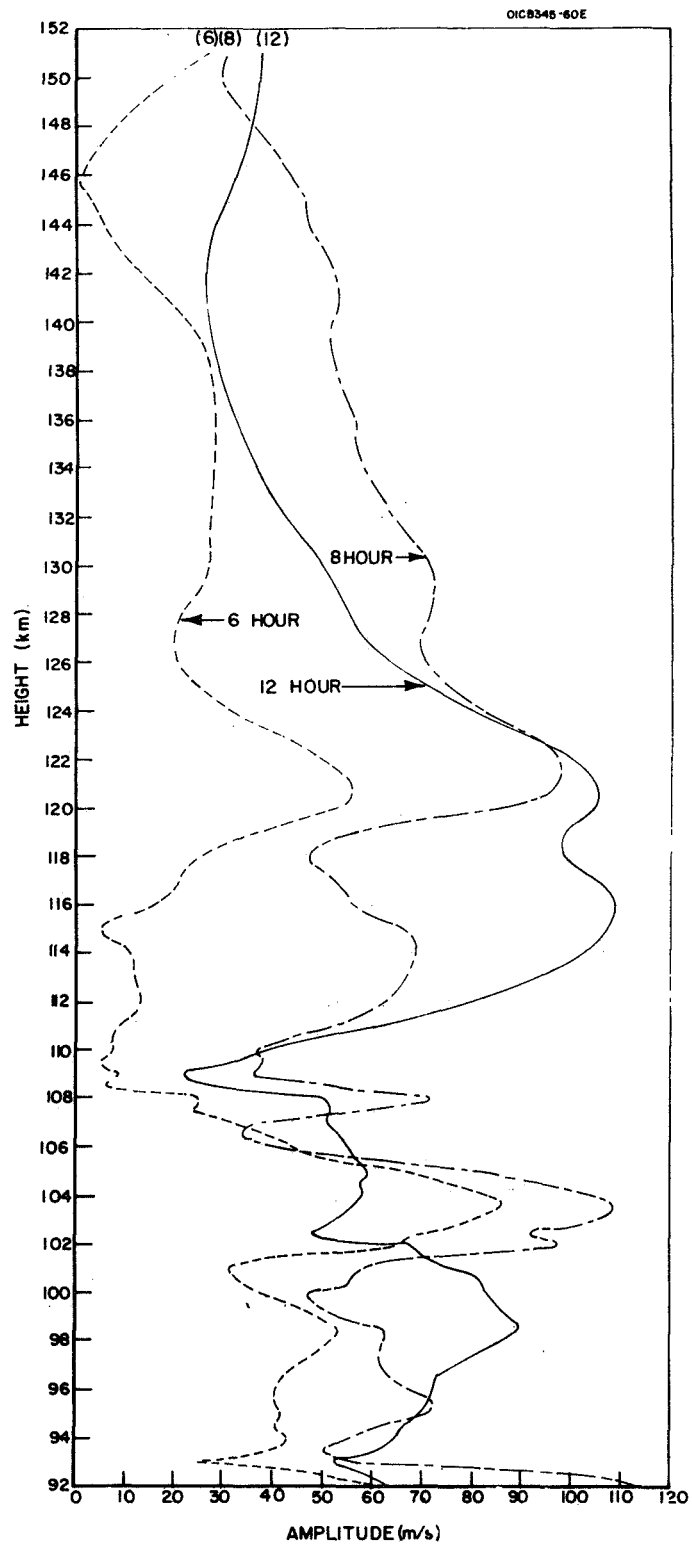


Figure 2. Amplitudes of the 12, k, and 6-hour components of the eastward wind.

Figures 3 and 4 are the altitude profiles of the phases and amplitudes, respectively, of the 12, 8, and 6-hour periodic components of the northward wind. All amplitudes are very low in the 127-130 km region (Figure 4), which also exhibits abrupt phase changes. Above this region and below it (down to 115 km) the phases vary slowly with altitude. The region 104-107 km also exhibits low amplitudes and abrupt phase changes for all three components. The region 107-115 km exhibits very large north-south shears for all seven observations. This is reflected in the unrealistically high amplitudes in this region, as shown in Figure 4. It is evident that the analytic representation of the data in this region is not physically meaningful.

Figure 5 is the altitude profile of the average (or "prevailing") eastward and northward winds. The wavelike appearance of both curves, and the fact that the curve for the eastward component is, essentially, the curve for the northward component displaced upward, imply strongly that the prevailing component is intimately related to a periodic component with a long period. In the region below 120 km, the curves appear to represent a wave with vertical wavelength of 20 ± 5 km, roughly the wavelength of the diurnal component at these altitudes.

Although these results consist of reasonable values for the amplitudes and phase variations of the harmonic components used, the analysis remains unsatisfactory because the characteristics of the diurnal component remain unresolved. If a 24-hour period is used among the three periods of the representation, the results show unreasonably large amplitudes and erratic variations with height. This is primarily due to the fact that there is no redundancy in the data (i.e., the number of parameters equals the number of observations). Thus, if the measurements contain small amounts of short-period components and/or small errors, these can cause a large change in the amplitude and phase of the slowly varying 24-hour component in the context of this method. One way to get around the difficulty is to assume (parametrically) a smooth variation of the amplitude and phase of each component with height. This method would then analyze the altitude and time characteristics of the winds simultaneously. The number of parameters involved would then be a small fraction of the number of data points. A least-squares determination of these parameters would then avoid the uncertainties involved in the current analysis. Further details of the proposed method are given in Section IV.

C. Statistical Analysis of All Data from Wallops Island

The wind data from 57 Wallops Island trails were analyzed statistically, and the results of this analysis were reported in the Final Report under Contract No. NAS5-11572. This sample of 57 trails contained 42 trails launched at either morning or evening twilight, and only 15 trails launched at various times during the night. Since that analysis was completed, 15 more trails were launched, 11 of them at night. The new, enlarged sample was subjected to a different type of analysis, which is essentially the same as the harmonic analysis described in the preceding part of this

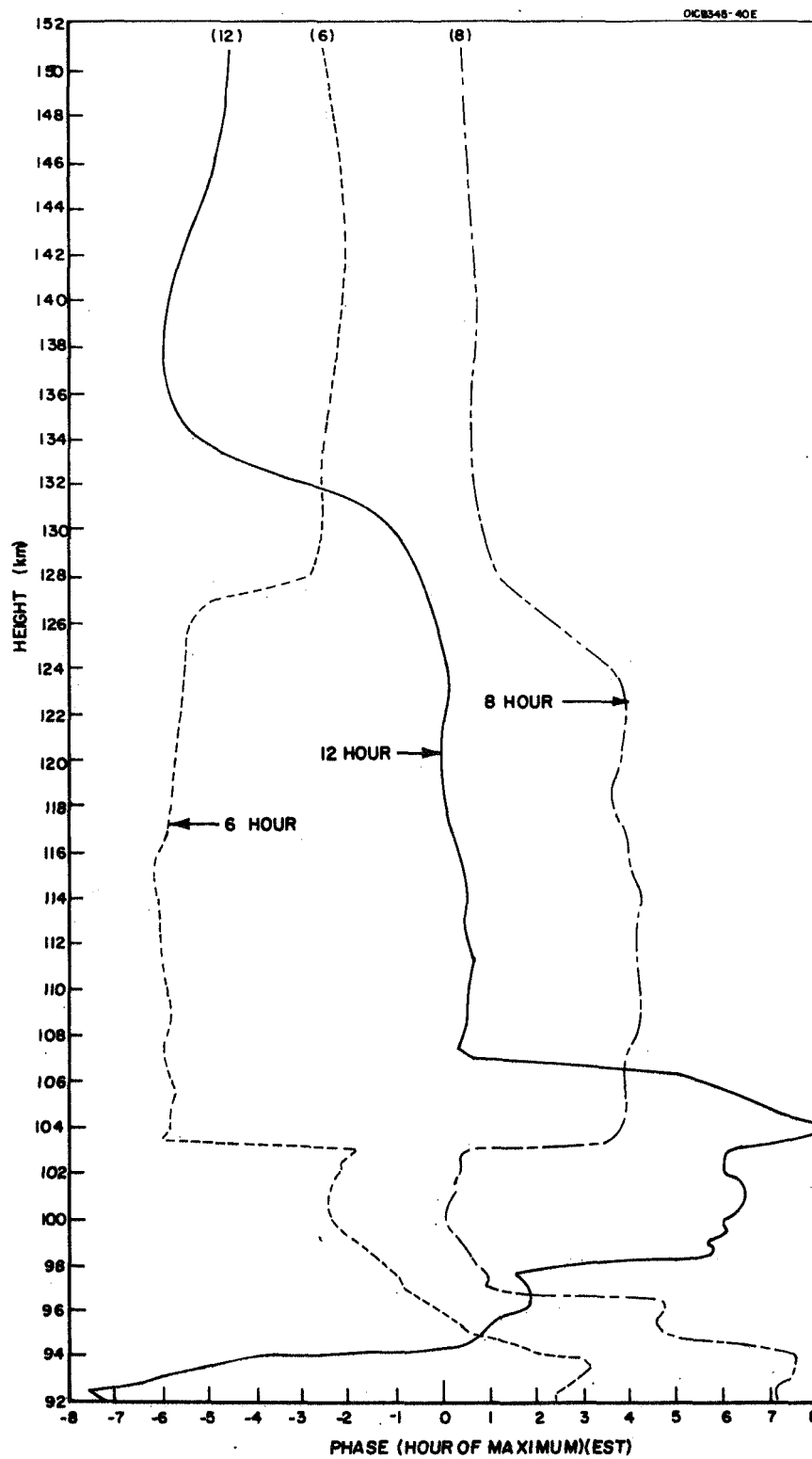


Figure 3. Phases of the 12, 8, and 6-hour components of the northward wind.

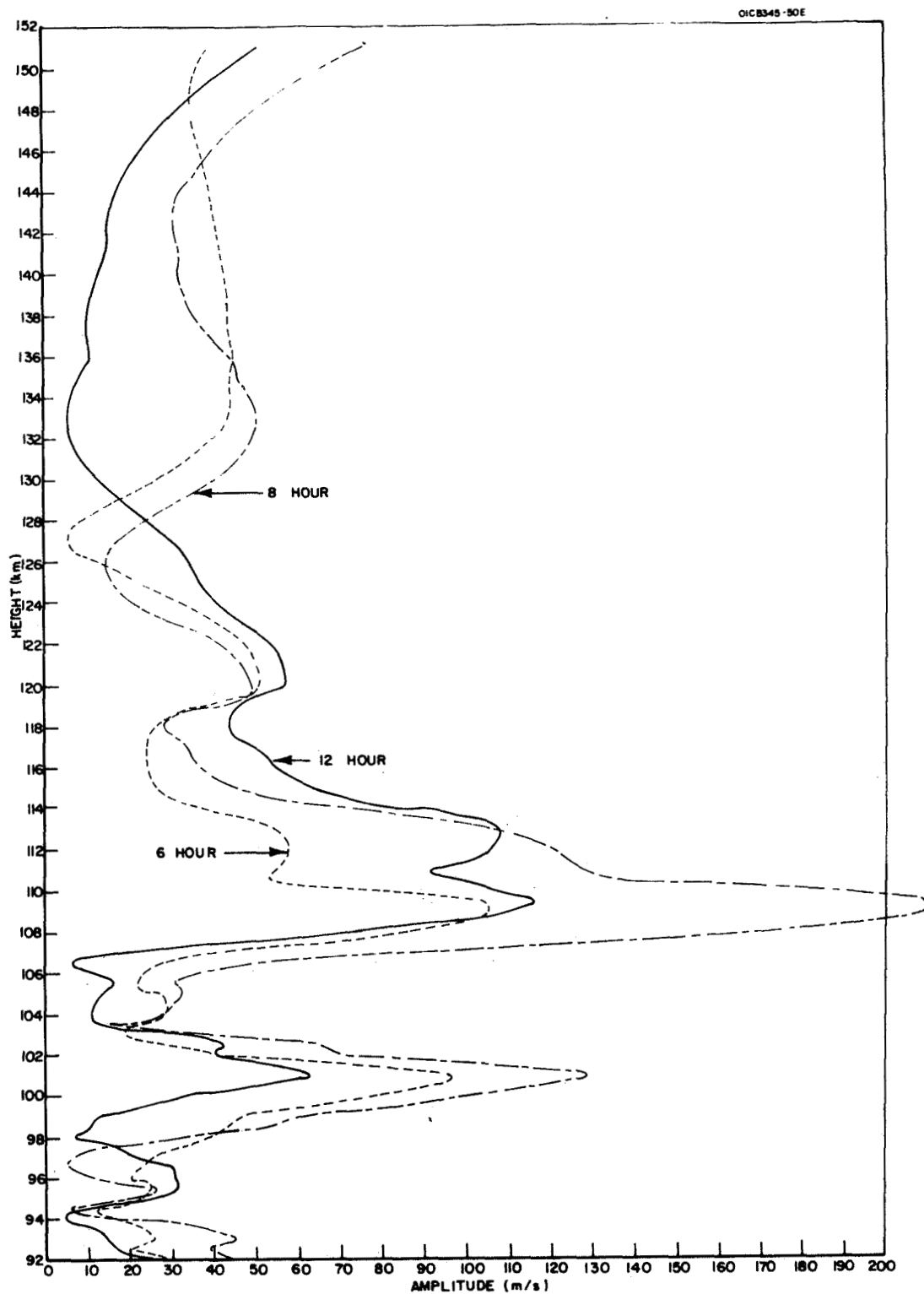


Figure 4. Amplitudes of the 12, 8, and 6-hour components of the northward wind.

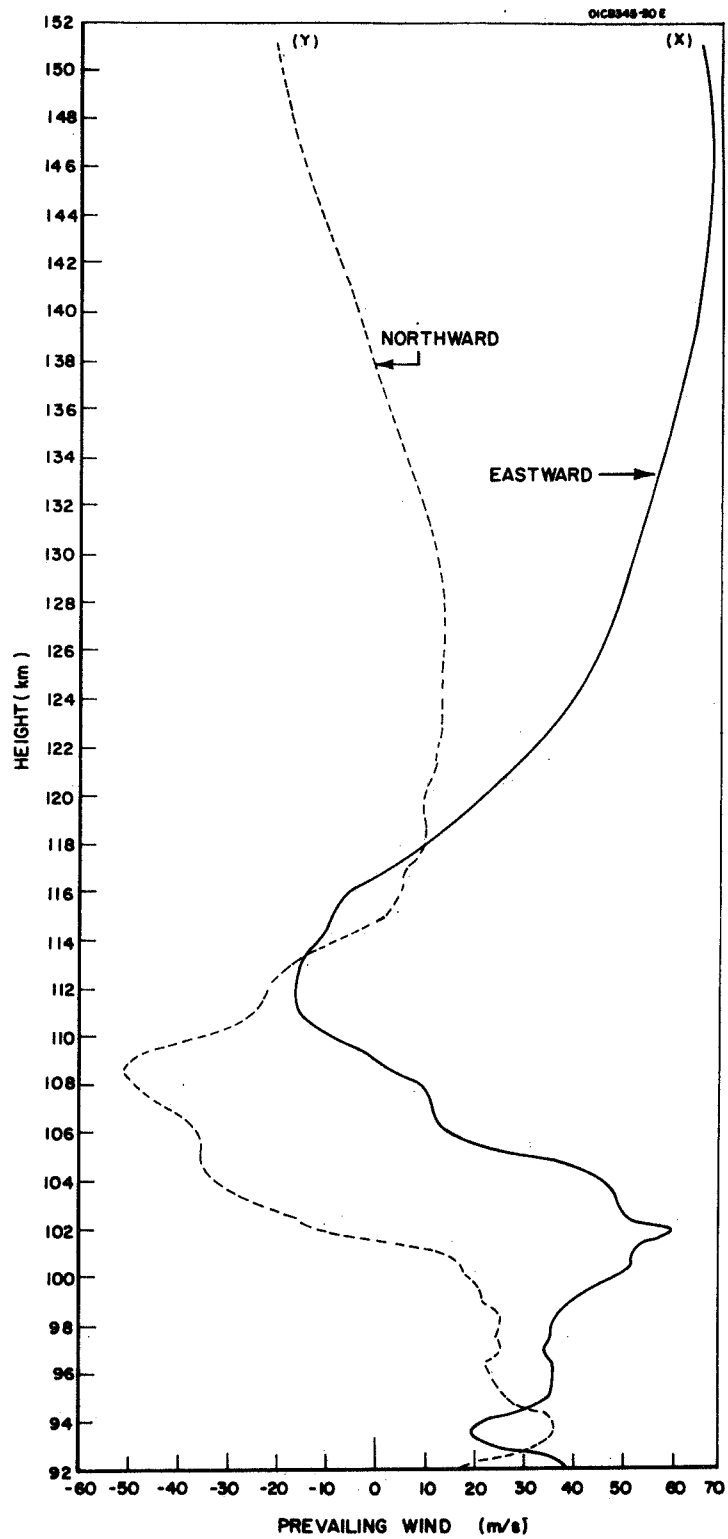


Figure 5. Amplitudes of the prevailing eastward and northward winds.

section. Specifically, the data which were accumulated over a period of 12 years were treated as if they were collected during a single night between 17:00 hours and 06:30 hours. Thus, treatment of the data effectively ignores day-to-day, seasonal and solar cycle variations. The results, accordingly, describe the temporal variation during the night in an average way for the sample as a whole. Some idea of the nature of the data, and the resulting representation of it can be gained from inspection of Figure 6. In this figure, which depicts the eastward wind at 100 km, the abscissa is labeled by the Wallops Island local time (EST), while the ordinate marks the eastward wind in meters per second. The crosses in the figure represent individual measurements. The curve represents a 7-parameter fit to the data, the parameters being amplitude and phase for periodic components with periods of 12, 8, and 6-hours as well as a parameter representing the prevailing wind. (Computer plots like Figure 6 have been made for both the eastward and northward components at 1 km intervals between 90 and 150 km).

The crosses in Figure 6 suggest the existence of maxima in the vicinity of 19:30 hours and 04:00 hours, and a minimum between 22:00 and 24:00 hours. There is no qualitative indication of a 24-hour component in this figure. This is not surprising, since the diurnal component has a relatively short wavelength (about 20 km) in this region, and hence, seasonal and solar-cycle variations would tend to change its phase randomly at any given height for a data sample like the present one. In summary, the presence of the diurnal variation would tend to be masked in this type of sample.

Figure 7 is again a plot of the eastward wind at 100 km. The heavy solid curve is the same as the curve of Figure 6. The other curves represent the six time series of measurements taken at Wallops Island. The curves connecting successive measurements of each series are hand-drawn. There is considerable variation among the different series, but each one has maxima at the twilight hours, and a minimum around 23:00, indicating strongly the presence of a semi-diurnal component.

Figures 8 through 14 represent the results of the harmonic analysis in the height range 92-151 km. The symbols X and Y on the curves denote eastward and northward, respectively. Because of the nature of the sample, the results cannot be taken as descriptive of the way the 12-hour (or 8 or 6-hour) component propagates in this part of the atmosphere. Rather, these graphs collectively represent the average night-time temporal characteristics of the horizontal wind components.

Figure 14 is a plot of the parameters C_0 for the eastward (X) and northward (Y) components. Like the corresponding figure for the data of February 1969, the curves of Figure 14 have a wavelike appearance, with the curve for the northward component generally displaced downward from the curve for the eastward component. As before, these "prevailing" components reflect the characteristics of the diurnal component.

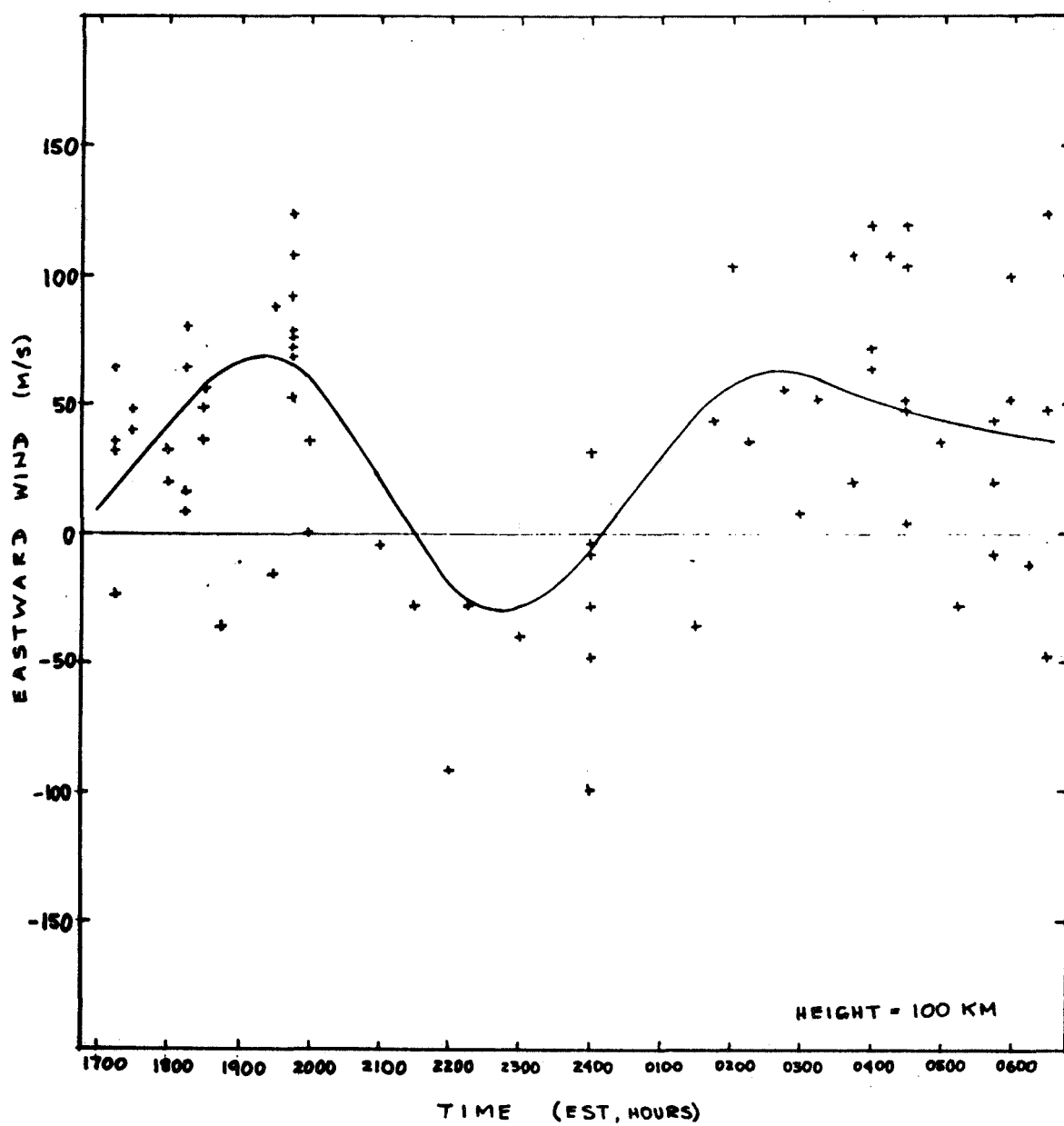


Figure 6. Summation of all eastward wind measurements at 100 km, made from Wallops Island.

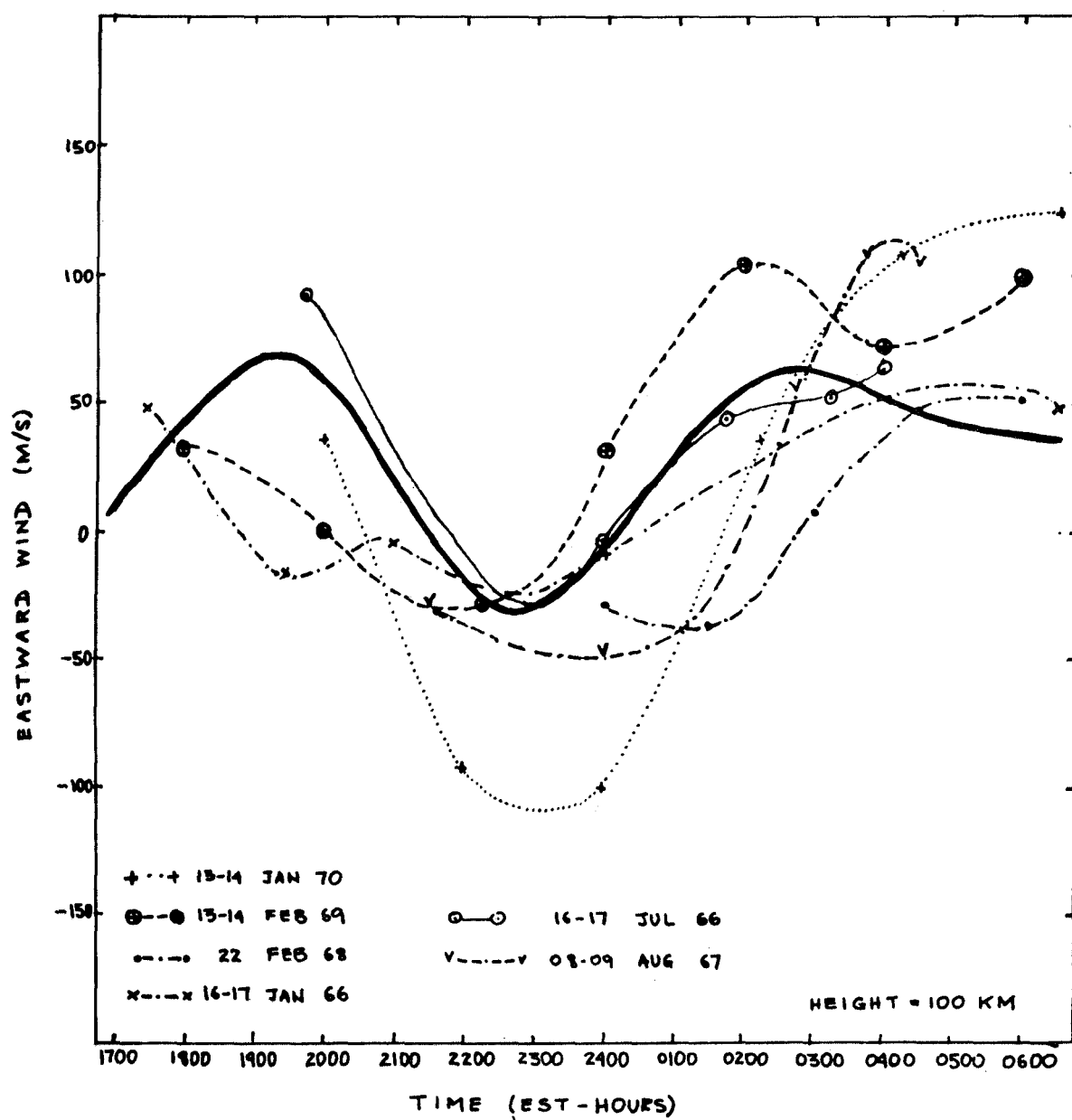


Figure 7. Summation of eastward winds at 100 km, obtained from time series at Wallops Island.

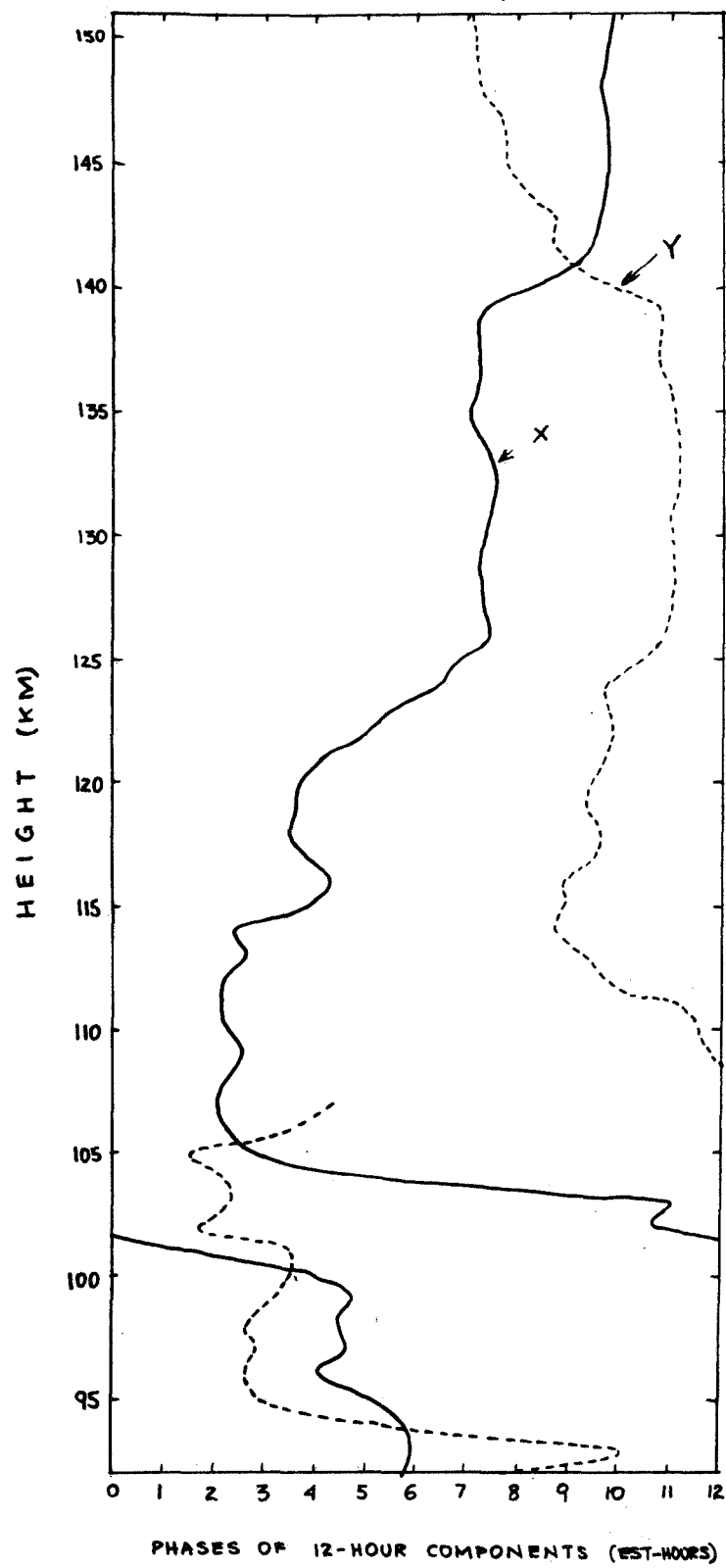


Figure 8

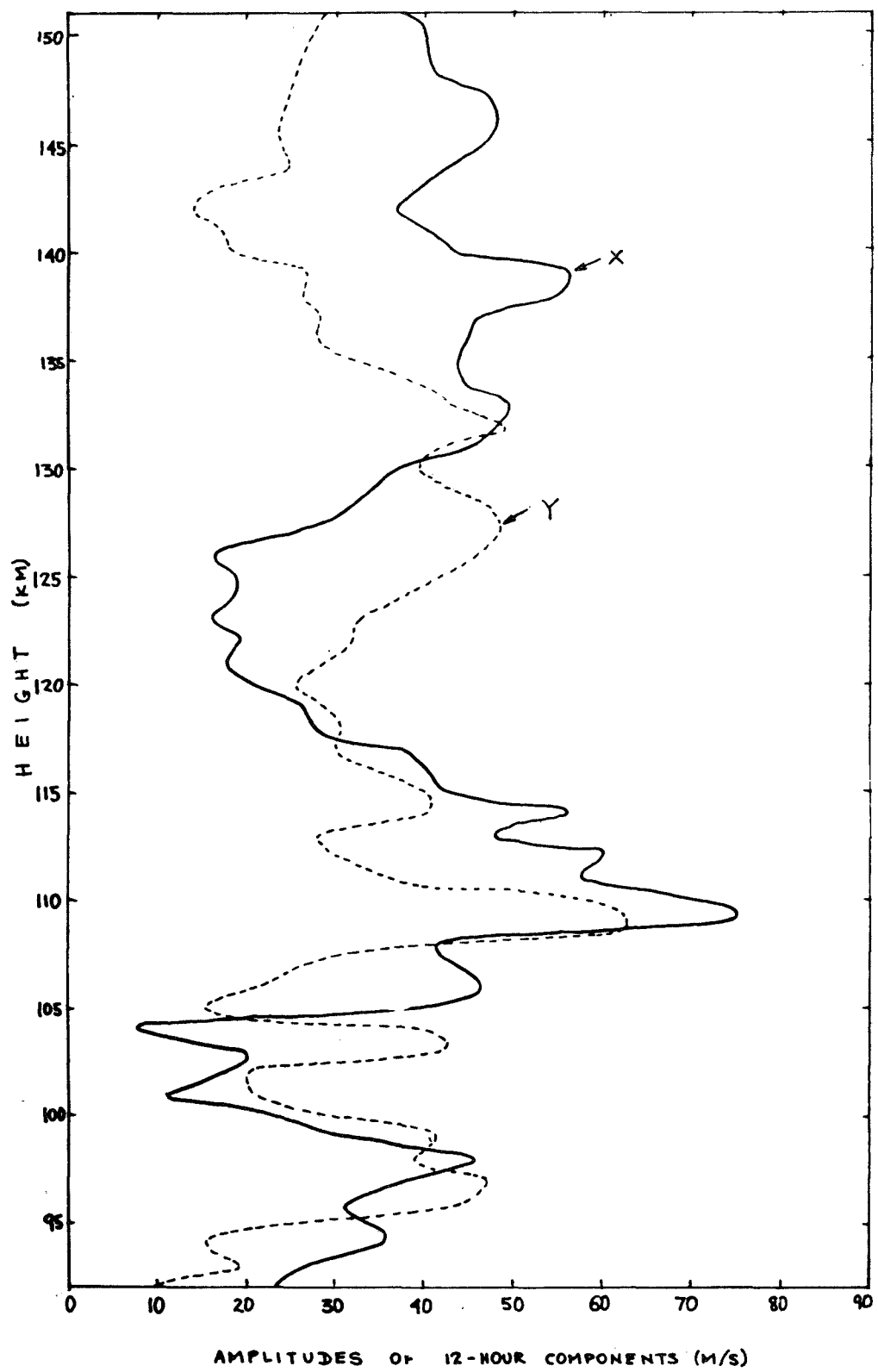


Figure 9

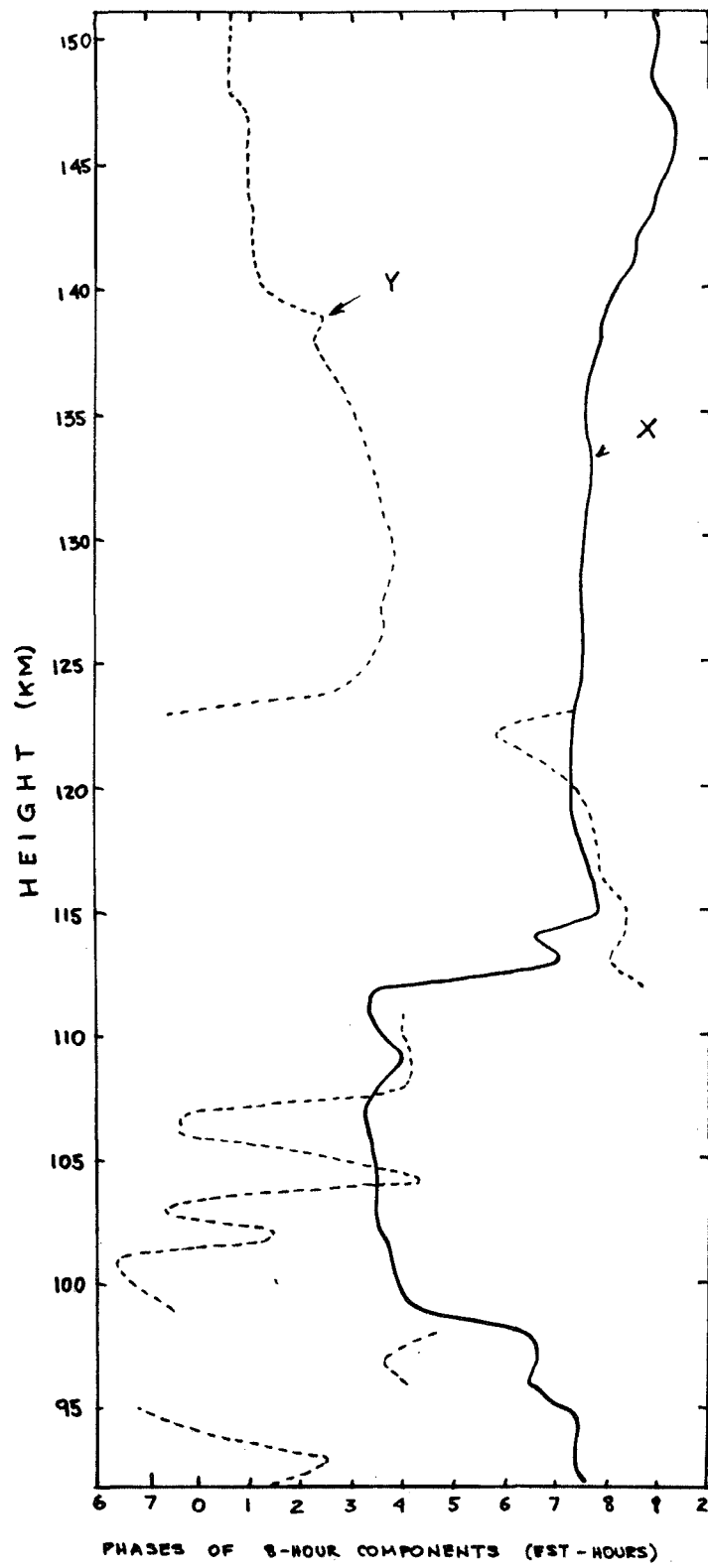


Figure 10

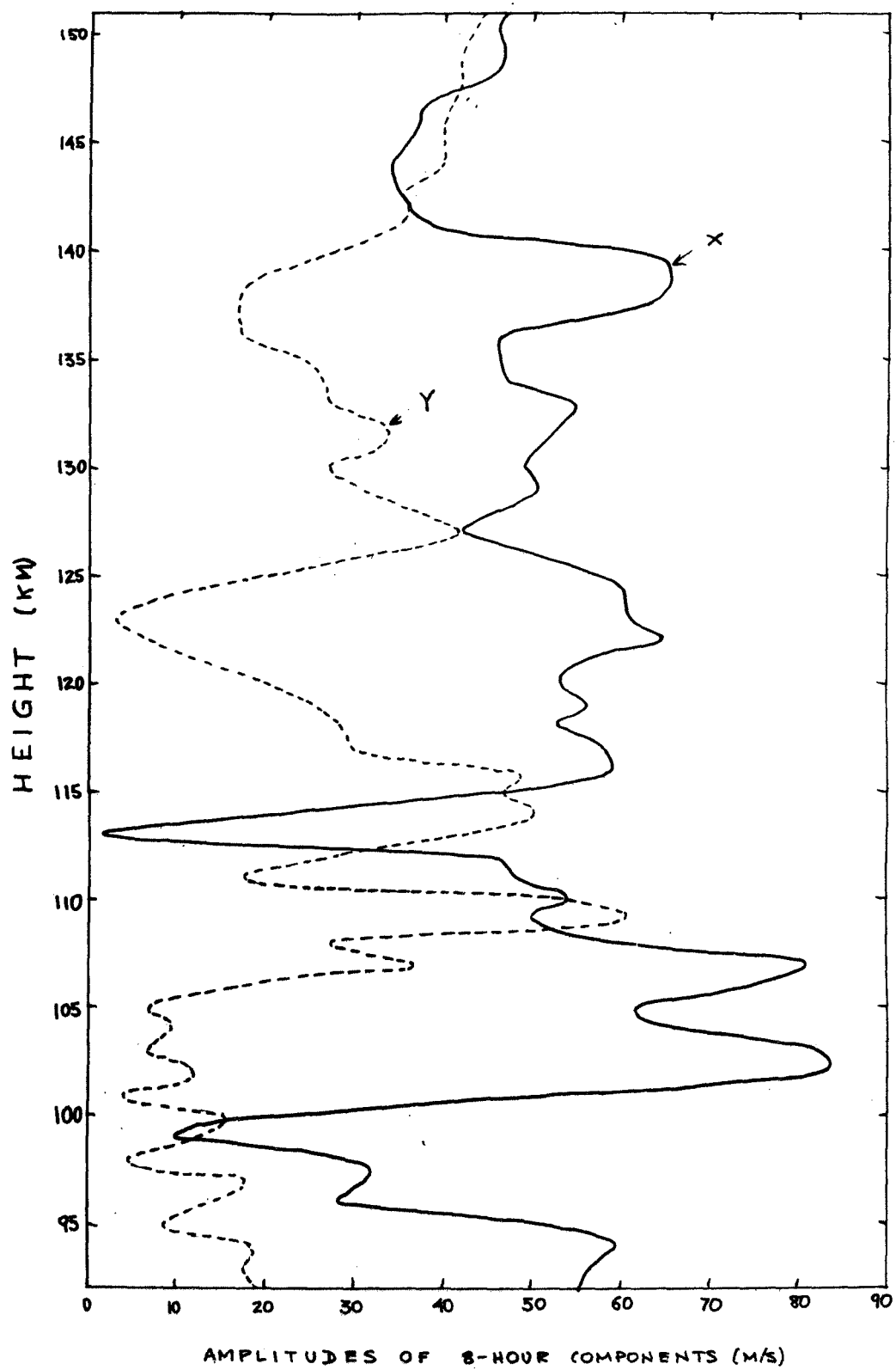


Figure 11

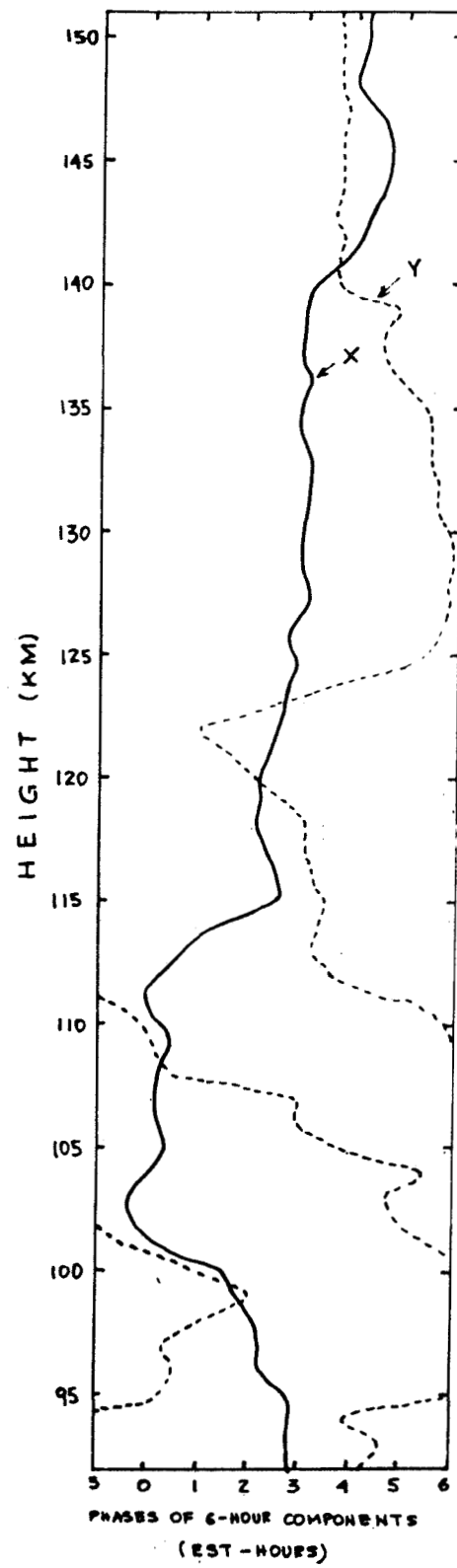


Figure 12

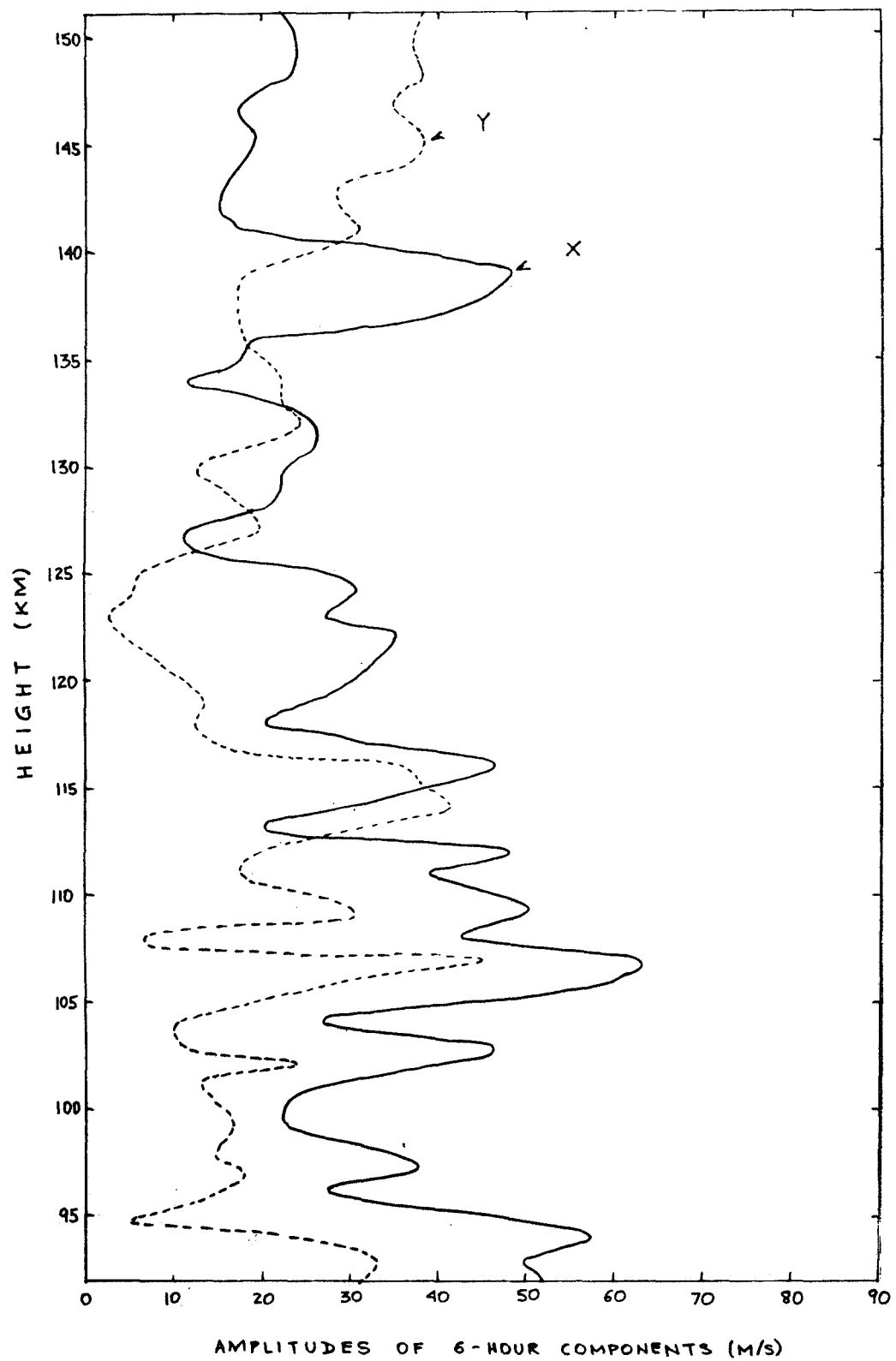


Figure 13

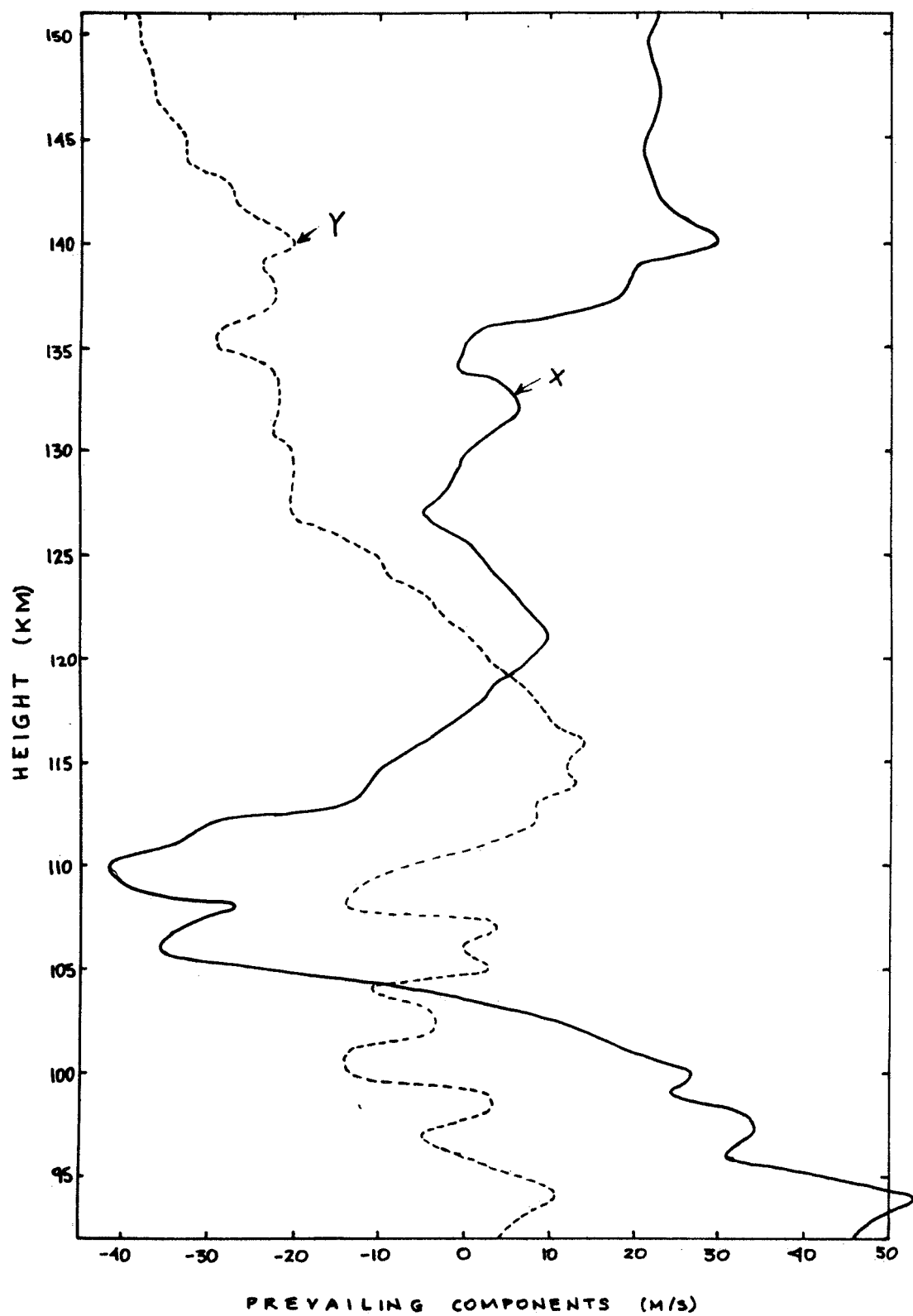


Figure 14. Amplitudes of the eastward (x) and northward (r) prevailing components.

SECTION III

THEORETICAL INVESTIGATIONS

A. Introduction

One of the objectives of this contract is the investigation of the physical properties of the atmosphere implied by the observed winds. To facilitate this investigation it is useful to construct a model appropriate to the observation site (Wallops Island, Va.). Accordingly, the equations pertinent to a spherical earth are approximated by a new set of equations appropriate to a plane-stratified atmosphere with origin at Wallops Island. From the new set of equations, formulas are derived which express the variation of the density, pressure, temperature, and heating rate as a function of the observable quantities, namely the eastward and northward wind components. Further, the new set of equations is so formulated as to allow the calculation of second-order quantities (in the sense of a perturbation theory). Second-order results arising from the main diurnal propagating mode are derived, including numerical estimates.

B. General Equations

The equations that govern atmospheric dynamics are:

$$\frac{d\rho}{dt} = -\rho \nabla \cdot \vec{u} \quad (1)$$

expressing the conservation of mass;

$$p = \rho T' R_g / M \quad (2)$$

relating pressure, density, and temperature;

$$\frac{dp}{dt} - \gamma \left(\frac{p}{\rho} \right) \frac{d\rho}{dt} + \frac{p}{M} \frac{dM}{dt} = (r-1) Q \quad (3)$$

expressing the first law of thermodynamics; and

$$\rho \frac{d\vec{u}}{dt} + \nabla p - \rho \vec{g} + \rho \vec{\omega}_c \times \vec{u} = \vec{F} \quad (4)$$

expressing the equations of motion, where:

ρ = density

\vec{u} = (u,v,w) = velocity field (eastward, northward, upward)

p = pressure

T' = temperature

R_G = universal gas constant

M = mean molecular weight

γ = ratio of specific heats

Q = heating rate (energy/unit time/unit volume)

\vec{g} = gravitational acceleration vector

$\vec{\omega}_c$ = coriolis vector

\vec{F} = force per unit volume

$\frac{d}{dt} = \frac{d}{dt} + \vec{u} \cdot \nabla$

The zero-order approximation describes the properties of the ambient atmosphere:

$$p_0 = \rho_0 T_0 R_G / M, \quad \vec{u}_0 = 0, \quad Q_0 = 0, \quad \frac{\partial p_0}{\partial z} = -\rho_0 g. \quad (5)$$

C. Formal Equations of First and Higher Order For a Plane-Stratified Atmosphere

The first-order (and higher-order) equations are conveniently written in terms of the variables P , R , T , defined by the equations:

$$\begin{aligned}
p &= p_0 (1 + P) & P &= P_1 + P_2 + \dots \\
\rho &= \rho_0 (1 + R) & R &= R_1 + R_2 + \dots \\
T &= T_0 (1 + T) & T &= T_1 + T_2 + \dots \\
\vec{u} &= \vec{u}_1 + \vec{u}_2 + \dots & & (6)
\end{aligned}$$

where the subscript specifies the order. For a plane-stratified atmosphere, the equations are:

$$P_n = R_n + T_n + G_{Tn} \quad (7)$$

$$\frac{\partial R_n}{\partial t} + w_n \left(\frac{1}{\rho_0} \frac{\partial \rho_0}{\partial z} \right) + \nabla \cdot \vec{u}_n = G_{Rn} \quad (8)$$

$$\frac{\partial u_n}{\partial t} - \bar{w} u_n + \left(\frac{\rho_0}{\rho_0} \right) \frac{\partial P_n}{\partial x} = F_{xn} / \rho_0 + G_{xn} \quad (9)$$

$$\frac{\partial v_n}{\partial t} + \bar{w} v_n + \left(\frac{\rho_0}{\rho_0} \right) \frac{\partial P_n}{\partial y} = F_{yn} / \rho_0 + G_{yn} \quad (10)$$

$$(\rho_0 / g \rho_0) \frac{\partial P_n}{\partial z} + R_n - P_n = F_{zn} / g \rho_0 \quad (11)$$

$$\frac{\partial P_n}{\partial t} - \gamma \frac{\partial R_n}{\partial t} + w_n \left(\frac{1}{\rho_0} \frac{\partial \rho_0}{\partial z} - \gamma \frac{1}{\rho_0} \frac{\partial \rho_0}{\partial z} + \frac{1}{\mu} \frac{\partial M}{\partial z} \right) = \frac{(\gamma-1)}{\gamma} Q_n + G_{Hn} \quad (12)$$

The coordinates x, y, and z are positive eastward, northward, and upward, respectively. In equation (11) the conventional approximation has been followed, the neglected terms being three orders of magnitude smaller than the terms retained. The G functions on the right of equations (7) through (12) are defined as follows:

$$G_T = R T \quad (13)$$

$$G_R = -w R \left(\frac{1}{\rho_0} \frac{\partial \rho_0}{\partial z} \right) - \vec{u} \cdot \nabla R - R \nabla \cdot \vec{u} \quad (14)$$

$$G_x = -R \frac{\partial u}{\partial t} - \vec{u} \cdot \nabla u + \bar{\omega} R v - R \vec{u} \cdot \nabla v \quad (15)$$

$$G_y = -R \frac{\partial v}{\partial t} - \vec{u} \cdot \nabla v - \bar{\omega} R u - R \vec{u} \cdot \nabla v \quad (16)$$

$$\begin{aligned} G_H = & -w \left[P \left(\frac{1}{\rho_0} \frac{\partial \rho_0}{\partial z} \right) - \gamma R \left(\frac{1}{\rho_0} \frac{\partial \rho_0}{\partial z} \right) + P \left(\frac{1}{M} \frac{\partial M}{\partial z} \right) - \vec{u} \cdot \nabla (P - \gamma R) \right. \\ & + \gamma (P - R) \left[\frac{\partial R}{\partial t} + w \left(\frac{1}{\rho_0} \frac{\partial \rho_0}{\partial z} \right) \right] \\ & \left. + \gamma \left[\frac{(1+P)}{(1+R)} - 1 - (P-R) \right] \left[\frac{\partial R}{\partial t} + w \left(\frac{1}{\rho_0} \frac{\partial \rho_0}{\partial z} \right) \right] + \gamma \left[\frac{(1+P)}{(1+R)} - 1 \right] \left[w R \left(\frac{1}{\rho_0} \frac{\partial \rho_0}{\partial z} \right) + \vec{u} \cdot \nabla R \right] \right] \quad (17) \end{aligned}$$

$$\bar{\omega} = 2 \Omega \sin \theta$$

$$\Omega = \text{rate of terrestrial rotation}$$

$$\theta = \text{latitude}$$

As can be seen from equations (13) through (17), all the G functions vanish to first order, since they are at least bilinear in the perturbation functions. The correspondence between the rectangular coordinates x and y, and the polar coordinates θ , ϕ (ϕ = longitude measured eastward) is expressed by the equations:

$$x = r_e \sin \theta \cdot \phi, \quad y = r_e \cdot \phi \quad (18)$$

where r_e is the radius of the earth. In the plane-stratified atmospheric model described above, the interdependence of x and y (through θ) is neglected. Further, the use of r_e in place of $(r_e + z)$ in (18) is an additional approximation. Consequently, the approximate model is valid only in the vicinity of the chosen origin, and for a finite height range.

D. Forces

The forces present in the atmosphere are due to ion drag and to viscosity. The force due to viscosity has the form:

$$\vec{F}_v = \nabla \cdot \vec{\sigma} \quad (19)$$

where $\vec{\sigma}$ is the tensor specified by:

$$\sigma_{ik} = \eta \left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i} \right) + \delta_{ik} \left(\zeta - \frac{2}{3}\eta \right) \nabla \cdot \vec{u} \quad (20)$$

where η and ζ are dynamic viscosity coefficients. In practice, because the vertical gradients are several orders of magnitude larger than horizontal gradients, the force term simplifies to the expression:

$$\vec{F}_v \approx \frac{\partial}{\partial z} \left(\eta \frac{\partial \vec{u}}{\partial z} \right) \quad (21)$$

The forces due to ion drag are generally taken to be:

$$F_{xI} = -\rho \cdot \sin \beta_x \cdot D \cdot u \quad F_{yI} = -\rho \cdot \sin \beta_y \cdot D \cdot v \quad (22)$$

where D is proportional to the ion density, and β_x and β_y are the angles between the magnetic field and the x and y axes, respectively.

E. Heating and Cooling

The heating term Q can be written as the sum of five terms:

$$Q = Q_J + Q_K + Q_V + Q_I + Q_C \quad (23)$$

where:

$$Q_J = \rho J = \text{heating due to absorption of insolation}$$

$$Q_K = \nabla \cdot (\kappa \nabla T') = \text{heating from thermal conduction}$$

$$Q_V = \frac{1}{2} \eta \sum_{i,k} \left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i} \right)^2 + \left(\zeta - \frac{2}{3}\eta \right) (\nabla \cdot \vec{u})^2$$

= heating from viscous dissipation

$$Q_C = -aT' = \text{cooling due to infrared radiation}$$

$$Q_I = \rho \cdot D \cdot (u^2 \sin \beta_x + v^2 \sin \beta_y)$$

= heating due to ion drag

$$\kappa = \text{coefficient of thermal conduction}$$

$$a = \text{cooling rate coefficient.}$$

The terms Q_V and Q_I are bilinear in the variables and, therefore, play no part in a first order theory. Because of the disparity between horizontal and vertical gradients, and because the horizontal velocity components are more than two orders of magnitude larger than the vertical component, the term Q_V simplifies to:

$$Q_V \cong \eta \left[\left(\frac{\partial u}{\partial z} \right)^2 + \left(\frac{\partial v}{\partial z} \right)^2 \right] \quad (24)$$

Similarly:

$$Q_K \cong \frac{\partial}{\partial z} \left(\kappa \frac{\partial T'}{\partial z} \right) \quad (25)$$

F. Determination of the Horizontal Wave-numbers For a Plane-Stratified Atmosphere

For a single tidal component of frequency σ (or period $2\pi/\sigma$) and horizontal wave-numbers k and ℓ in the x (eastward) and y (northward) directions. The dependence on t , x , and y is expressed by:

$$(u, v, w, P, R, T) = e^{i(\sigma t + kx + \ell y)}. \quad (26)$$

The Equations (7) through (12) become:

$$\begin{aligned} P_1 &= R_1 + T_1 \\ i\sigma R_1 + w_1 \left(\frac{1}{\rho_0} \frac{\partial \rho_0}{\partial z} \right) + ik\mu_1 + i\ell v_1 + \frac{\partial w_1}{\partial z} &= 0 \\ i\sigma \mu_1 - \bar{\omega} v_1 + (\rho_0/\rho_0) ikP_1 &= -\mu_1 D \sin \beta_x + \frac{1}{\rho_0} \frac{\partial}{\partial z} \left(\eta \frac{\partial \mu_1}{\partial z} \right) \\ i\sigma v_1 + \bar{\omega} \mu_1 + (\rho_0/\rho_0) i\ell P_1 &= -v_1 D \sin \beta_y + \frac{1}{\rho_0} \frac{\partial}{\partial z} \left(\eta \frac{\partial v_1}{\partial z} \right) \\ (\rho_0/\rho_0) \frac{\partial P_1}{\partial z} &= P_1 - R_1 \\ i\sigma (P_1 - \gamma R_1) + w_1 \left(\frac{1}{\rho_0} \frac{\partial \rho_0}{\partial z} - \gamma \frac{1}{\rho_0} \frac{\partial \rho_0}{\partial z} + \frac{1}{M} \frac{\partial M}{\partial z} \right) &= \frac{(k-1)}{\rho_0} Q_1 \end{aligned}$$

For the present purposes, it suffices to consider a simplified atmosphere, for which:

$$\eta = 0, \quad Q_1 = 0, \quad D = 0, \quad (33)$$

and T_0 and M are constant. Then:

$$\frac{1}{p_0} \frac{\partial p_0}{\partial z} = \frac{1}{p_0} \frac{\partial p_0}{\partial z} = -\frac{1}{H}, \quad H = R_g T_0 / g M \quad (34)$$

and the equations simplify to:

$$P_1 = R_1 + T_1$$

$$i\tau R_1 + ik\mu_1 + i\ell\tau_1 - \frac{w_1}{H} + \frac{\partial w_1}{\partial z} = 0$$

$$i\tau\mu_1 - \bar{w}\tau_1 + ikgHP_1 = 0$$

$$i\tau\tau_1 + \bar{w}\mu_1 + i\ell gHP_1 = 0$$

$$H \frac{\partial P_1}{\partial z} = P_1 - R_1$$

$$i\tau(P_1 - \gamma R_1) + \frac{(r-1)}{H} w_1 = 0$$

The solutions of these equations are:

$$\mu_1 = \frac{(-\sigma k + i \bar{\omega} \ell)}{(r^2 - \bar{\omega}^2)} g H P_1$$

$$\nu_1 = \frac{(-\sigma \ell - i \bar{\omega} k)}{(r^2 - \bar{\omega}^2)} g H P_1$$

$$w_1 = i \sigma H \left[P_1 - \frac{\delta}{(\delta-1)} H \frac{\partial P_1}{\partial z} \right]$$

$$R_1 = P_1 - H \frac{\partial P_1}{\partial z}$$

and P is the solution of the differential equation:

$$H^2 \frac{\partial^2 P_1}{\partial z^2} - H \frac{\partial P_1}{\partial z} + \frac{(\delta-1)}{\delta} \frac{(k^2 + \ell^2)}{(r^2 - \bar{\omega}^2)} g H P_1 = 0 \quad (45)$$

To achieve correspondence with tidal theory we must set:

$$\frac{g (k^2 + \ell^2)}{(r^2 - \bar{\omega}^2)} = \frac{1}{h} \quad (46)$$

where h is the eigenvalue (or "equivalent depth") of tidal theory appropriate to the tidal mode under consideration. If n is defined as the zonal wave-number of tidal theory, then k must be chosen as:

$$k = n / r_e \sin \theta \quad (47)$$

where r_e is the radius at the earth, and θ is the latitude. The meridional wave-number is then determined from Equation (46). Because the

latitude enters into the determination of k and ℓ , the validity of the approximate plane-stratified model is limited to the vicinity of the latitude used to determine the values of k and ℓ .

G. First-Order Solutions For The Main Diurnal Propagating Mode

Of the tidal modes known to have appreciable amplitudes in the upper atmosphere, the main diurnal propagating mode has the shortest vertical wavelength. Accordingly, it is reasonable to assume that it is the primary contributor to non-linear interactions. The purpose of the calculation presented here is to examine the magnitude of the second-order terms that arise from the main diurnal propagating mode.

Above 80 km, there is very little heating of the atmosphere. Also, below 100 km the terms due to viscosity and conductivity are quite small (this is quantitatively verified by the calculation). Finally, in this altitude range, the atmosphere is nearly isothermal. Hence, the present calculation is quantitatively significant between 80 and 100 km.

From the two solutions of Equation (45), the one which propagates energy upward is chosen. In order to facilitate the numerical computations it is convenient to write:

$$P_1 = \hat{P}_1 e^{i\ell y + z/z_H + i\alpha} \quad \alpha \equiv \sigma t + kx + qz/H \quad (48)$$

$$R_1 = \hat{R}_1 P_1 \quad \hat{R}_1 \equiv \frac{1}{2} - iq \quad (49)$$

$$u_1 = \hat{u}_1 P_1 \quad \hat{u}_1 \equiv gH \frac{(-\sigma k + i\bar{\omega}\ell)}{(\sigma^2 - \bar{\omega}^2)} \quad (50)$$

$$v_1 = \hat{v}_1 P_1 \quad \hat{v}_1 \equiv gH \frac{(-\sigma\ell - i\bar{\omega}k)}{(\sigma^2 - \bar{\omega}^2)} \quad (51)$$

$$w_1 = \hat{w}_1 P_1 \quad \hat{w}_1 \equiv \frac{\sigma H}{(\gamma - 1)} [\gamma q - i(1 - \gamma/2)] \quad (52)$$

$$T_1 = \hat{T}_1 P_1 \quad \hat{T}_1 \equiv \frac{1}{2} + iq \quad (53)$$

$$q^2 \equiv -\frac{1}{4} + \frac{(\gamma - 1)}{\gamma} \frac{(k^2 + \ell^2)}{(\sigma^2 - \bar{\omega}^2)} gH \quad (54)$$

The numerical values of the parameters used in the calculation are:

$$\begin{aligned}
 r_e &= 6.46 \times 10^6 \text{ m} & \ell &= \pm \text{ im} \\
 H &= 7.61 \times 10^3 \text{ m} & m &= 6.82 \times 10^{-7} \text{ m}^{-1} \\
 \theta &= 37^\circ 50' & g &= 9.52 \text{ m sec}^{-2} \\
 \bar{\omega} &= 2 \omega_e \sin \theta = 0.892 \text{ sec}^{-1} \times 10^{-4} & \sigma &= 0.727 \times 10^{-4} \text{ sec}^{-1} \\
 k &= 1/(r_e \sin \theta) = 2.52 \times 10^{-7} \text{ m}^{-1} & \gamma &= 1.4 \\
 & & \eta &= 1.692
 \end{aligned}$$

It will be noticed that the meridional wave-number ℓ is imaginary at this latitude. As will be shown below, the mode corresponding to the positive root $\ell = \text{im}$ by itself accounts for most (about 96 percent) of the meridional variation of the main semi-diurnal mode at this latitude. The relative magnitudes of the variables are:

$$\begin{aligned}
 \hat{R}_1 &= 0.5 - 1.692 i \quad (\text{dimensionless}) \\
 \hat{T}_1 &= 0.5 + 1.692 i \quad (\text{dimensionless}) \\
 \hat{W}_1 &= 3.278 - 0.415 i \text{ m sec}^{-1} \\
 \hat{u}_{1+} &= 2.148 \times 10^3 \text{ m sec}^{-1} \text{ for } \ell = + \text{ im} \\
 \hat{u}_{1-} &= -1.152 \times 10^3 \text{ m sec}^{-1} \text{ for } \ell = - \text{ im} \\
 \hat{v}_{1+} &= 1.956 \times 10^3 i \text{ m sec}^{-1} \text{ for } \ell = + \text{ im} \\
 \hat{v}_{1-} &= -0.734 \times 10^3 i \text{ m sec}^{-1} \text{ for } \ell = - \text{ im}
 \end{aligned}$$

Work performed under Contract NAS5-21513 indicates that the amplitude of the zonal wind over Wallops Island, Virginia at 100 km is approximately 75 meters per second for the main diurnal propagating mode. Using this value we can calculate the absolute values of the other variables at this altitude. These are presented in the following Table.

TABLE 2
ESTIMATED ABSOLUTE AMPLITUDES AND RELATIVE PHASES AT 100 km

| Variable | Amplitude | Relative Phase |
|----------|---------------------------|----------------|
| u_{1+} | 75.0 m sec ⁻¹ | 0.00 hours |
| v_{1+} | 68.3 m sec ⁻¹ | -6.00 |
| w_1 | 11.4 cm sec ⁻¹ | 0.48 |
| P_1 | 0.035 | 0.00 |
| R_1 | 0.062 | -4.90 |
| T_1 | 0.062 | 4.90 |

The phase of each variable is given as the hour at which the variable achieves its maximum relative to the hour of maximum for the pressure. The amplitudes for P, R, T are expressed as fractions of the ambient values. For the temperature, the listed fraction corresponds to an amplitude of 16°K.

It can be seen that the assumption that the vertical wind is more than two orders of magnitude smaller than the horizontal winds is born out by the computation. For the present model it is also possible to demonstrate that the force due to viscosity is small compared to the term involving the time derivative. Thus:

$$\begin{aligned}
 \left| \frac{\eta}{\rho_0} \frac{\partial^2 u}{\partial z^2} \right| / \left| \frac{\partial u}{\partial t} \right| &= \frac{\eta}{\rho_0 \sigma H^2} \left| i(q^2 - \frac{1}{4}) + q \right| \\
 &= |4.36 + 6.71 i| \times 10^{-3} \quad \text{at 95 km} \\
 &= |1.11 + 1.71 i| \times 10^{-2} \quad \text{at 100 km} \\
 &= |2.80 + 4.31 i| \times 10^{-2} \quad \text{at 105 km} \\
 &= |6.57 + 10.1 i| \times 10^{-2} \quad \text{at 110 km}
 \end{aligned}$$

The complete solution for the main propagating diurnal mode is the sum of the two solutions corresponding to $+\ell (= im)$ and $-\ell (= -im)$, as mentioned above. Using values obtained from graphs of the results (Lindzen, 1967) of tidal theory we calculate the ratio:

$$\frac{\hat{u}}{\hat{v}} = \frac{\hat{u}_+ + \hat{u}_-}{\hat{v}_+ + \hat{v}_-}$$

and thus, the ratio:

$$\hat{p}_- / \hat{p}_+ = - 0.041.$$

Thus, the solution corresponding to $+l$ accounts for 96 percent of the total solution, as stated previously.

Again using graphs from the same article we find:

$$\begin{aligned} \frac{1}{u} \frac{\partial u}{\partial y} &= - 0.54 \times 10^{-7} \text{ m}^{-1} \quad \text{at } \theta = 38^\circ \quad (\text{tidal theory}) \\ \frac{1}{v} \frac{\partial v}{\partial y} &= - 0.63 \times 10^{-7} \text{ m}^{-1} \quad \text{at } \theta = 38^\circ \quad (\text{tidal theory}) \end{aligned}$$

The same quantities computed on the basis of the approximate plane-stratified model employed here, are $- 0.65 \times 10^{-7} \text{ m}^{-1}$ and $- 0.66 \times 10^{-7} \text{ m}^{-1}$, respectively. Thus, the choice of meridional wavenumbers dictated by the plane-stratified model, Equation (46), gives quite satisfactory results.

H. Second-Order Solution For The Main Diurnal Propagating Mode

In second order, the main diurnal propagating mode gives rise to a semi-diurnal mode and to a "constant" (i.e., a mode with no dependence on t or x , and with only an amplitude variation in the vertical direction). Only the second-order terms arising from the mode corresponding to $+il$ ($= -m$) are computed here, since this mode accounts for 96 percent of the first-order solution. Formally, the second-order equations are:

$$i\tau_2 u_2 - \bar{w} v_2 + ik_2 g H P_2 = G_{x2} \quad (55)$$

$$i\tau_2 v_2 + \bar{w} u_2 - m_2 g H P_2 = G_{y2} \quad (56)$$

$$i\tau_2 R_2 - \frac{w_2}{H} + (ik_2 u_2 - m_2 v_2 + \frac{\partial w_2}{\partial z}) = G_{R2} \quad (57)$$

$$i\tau_2 (P_2 - \gamma R_2) + \frac{(\gamma-1)}{H} w_2 = G_{H2} \quad (58)$$

$$R_2 = P_2 - H \frac{\partial P_2}{\partial z} \quad (59)$$

$$T_2 = P_2 - R_2 - R_1 T_1 \quad (60)$$

where $k_2 = 2k$, $\sigma_2 = 2\sigma$, $m_2 = 2m$ for the semi-diurnal term.

The solution of these equations is straightforward. The detailed calculation is somewhat lengthy, and it is omitted. Numerically, the results for the semi-diurnal second-order term are:

$$\begin{aligned} P_2 &= \hat{P}_2 P_1^2 & \hat{P}_2 &= (0.1014 + 0.8893 i) \\ u_2 &= \hat{u}_2 P_1^2 & \hat{u}_2 &= (1.692 + 4.275 i) \times 10^3 \\ v_2 &= \hat{v}_2 P_1^2 & \hat{v}_2 &= (-3.808 + 1.658 i) \times 10^3 \\ w_2 &= \hat{w}_2 P_1^2 & \hat{w}_2 &= (-0.760 + 7.922 i) \\ R_2 &= \hat{R}_2 P_1^2 & \hat{R}_2 &= (3.009 - 0.343 i) \\ T_2 &= \hat{T}_2 P_1^2 & \hat{T}_2 &= (-4.464 + 1.232 i) \end{aligned}$$

Using the estimated value of 75 m/s for the eastward wind at 100 km we can calculate the absolute values and relative phases for the second-order semi-diurnal term. The results are presented in the following Table.

TABLE 3
ESTIMATED ABSOLUTE AMPLITUDES AND RELATIVE PHASES
AT 100 KM FOR THE SECOND-ORDER SEMI-DIURNAL SOLUTION

| Variable | Amplitude | Relative Phase | Relative Amp |
|----------|----------------------|----------------|--------------|
| u_{2+} | 5.6 m/s | - 2.28 hrs. | 7.5% |
| v_{2+} | 5.1 m/s | - 5.21 | 7.5% |
| w_{2+} | 1.0 cm/s | - 3.18 | 8.5% |
| P_{2+} | 1.1×10^{-3} | - 2.79 | 3.1% |
| R_{2+} | 3.7×10^{-3} | 0.22 | 6.0% |
| T_{2+} | 5.6×10^{-3} | - 5.48 | 9.0% |

The relative phase listed for each variable is the hour of maximum for that variable relative to the hour of maximum of the diurnal pressure variation P_1 . Also included in the Table for each variable is the ratio of the second-order (semi-diurnal) amplitude to the first-order (diurnal) amplitude. These ratios range from 3.1 percent for the pressure to 9.0 percent for the temperature.

Of primary interest in the present context is whether the non-linear, second-order terms contribute appreciably to the observed winds. The results indicate that at 100 km the second-order contributions amount to 5 to 6 meters per second. Although this value is a small fraction (7.5 percent) of the diurnal wind, it is certainly large enough to be observable, and it must therefore be born in mind during the analysis of observations. In the absence of viscosity, all variables grow exponentially with height, but whereas the first-order variables grow as $\exp(z-z_0)/2H$, the second-order variables grow twice as fast. Thus, it would follow that at 110 km the second-order solution would amount to 16 percent of the first-order solution at that height or 20 meters per second. Whether the second-order winds achieve this value at 110 km is uncertain, however, because viscous dissipation begins to play an important part above 100 km. This uncertainty can be resolved only by a detailed theoretical calculation or by a detailed analysis of the observations.

I. Formulas For The Analysis of Observations

In the expectation that the observed winds can be resolved into distinct tidal modes, it is of interest to derive formulas for the other atmospheric variables in terms of the observed variables. (This expectation seems to be borne out by the preliminary results of work performed under Contract NAS5-21513.) To this end, Equations (7) through (12) of Part 3 are rewritten in a form convenient for the present discussion:

$$T = P - R \quad (61)$$

$$\frac{\partial R}{\partial t} + \left(\frac{1}{\rho_0} \frac{\partial \rho_0}{\partial z}\right) w + \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (62)$$

$$\frac{\partial u}{\partial t} - \bar{\omega} v + gH \frac{\partial P}{\partial x} = -\sin \beta_x \cdot D \cdot u + \frac{1}{\rho_0} \frac{\partial}{\partial z} \left(\eta_0 \frac{\partial u}{\partial z} \right) \quad (63)$$

$$\frac{\partial v}{\partial t} + \bar{\omega} u + gH \frac{\partial P}{\partial y} = -\sin \beta_y \cdot D \cdot v + \frac{1}{\rho_0} \frac{\partial}{\partial z} \left(\eta_0 \frac{\partial v}{\partial z} \right) \quad (64)$$

$$R = P - H \frac{\partial P}{\partial z} \quad (65)$$

$$\frac{\partial P}{\partial t} - \gamma \frac{\partial P}{\partial t} + w \left(\frac{1}{\rho_0} \frac{\partial \rho_0}{\partial z} - \gamma \frac{1}{\rho_0} \frac{\partial \rho_0}{\partial z} + \frac{1}{M} \frac{\partial M}{\partial z} \right) = \frac{(g-1)}{P} Q \quad (66)$$

In these equations all the variables pertain to the same tidal mode. Further, the variables appear in their real (as opposed to complex) form. The purpose of the present discussion is to indicate how one can obtain the values of the variables P, R, T, w, and Q from the values of u and v deduced from the resolution of the observed winds into tidal modes. In addition, the quantity D, which is proportional to the local ion density, can be obtained. The steps involved in the analysis are indicated below.

$$\begin{aligned} \frac{\partial}{\partial y} \left[\frac{\partial u}{\partial t} - \bar{w}v - \frac{1}{\rho_0} \frac{\partial}{\partial z} (\eta_0 \frac{\partial u}{\partial z}) \right] - \frac{\partial}{\partial x} \left[\frac{\partial v}{\partial t} + \bar{w}u - \frac{1}{\rho_0} \frac{\partial}{\partial z} (\eta_0 \frac{\partial v}{\partial z}) \right] = \\ = D \left[-\sin \beta_x \frac{\partial u}{\partial y} + \sin \beta_y \frac{\partial v}{\partial x} \right] \end{aligned}$$

yields the value of D. If ion drag affects the wind velocity significantly, the value of D calculated from this formula will be significant. Thus, an estimate can be obtained of the local ion density and compared with concurrent measurements of the ion density obtained by other methods (e.g., electron probes, ionosondes) when available:

$$\begin{aligned} v \sin \beta_y \left[\frac{\partial u}{\partial t} - \bar{w}v - \frac{1}{\rho_0} \frac{\partial}{\partial z} (\eta_0 \frac{\partial u}{\partial z}) \right] - u \sin \beta_x \left[\frac{\partial v}{\partial t} + \bar{w}u - \frac{1}{\rho_0} \frac{\partial}{\partial z} (\eta_0 \frac{\partial v}{\partial z}) \right] = \\ = gH \left[u \sin \beta_x \frac{\partial P}{\partial y} - v \sin \beta_y \frac{\partial P}{\partial x} \right] \end{aligned}$$

yields the value of P. In this, as in the preceding formula, it is assumed that the horizontal wave numbers k and l have been established for the mode under consideration by correspondence with tidal theory, as prescribed in Part 6 of this section. Thus, the horizontal derivatives of P can be expressed in terms of P, and the last equation effectively serves to determine P. The density and temperature variations are determined from P by means of the equations:

$$R = P - H \frac{\partial P}{\partial z} \qquad T = P - R$$

The equation expressing the conservation of mass can be integrated to yield the result:

$$\begin{aligned} w(z) = \exp \left[- \int_{z_0}^z dz' \left(\frac{1}{\rho_0} \frac{\partial \rho_0}{\partial z'} \right) \right] * \left\{ w(z_0) \right. \\ \left. - \int_{z_0}^z dz' \left[\frac{\partial R}{\partial t} + \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right] * \exp \left[\int_{z_0}^{z'} dz'' \left(\frac{1}{\rho_0} \frac{\partial \rho_0}{\partial z''} \right) \right] \right\} \end{aligned}$$

At this time it is not entirely clear how the integration constant $w(z_0)$ should be chosen. Hence, further consideration of this question had best be postponed until these formulas are applied to a particular case. The net heating rate is obtained from the expression:

$$Q = \frac{p_0}{(\gamma-1)} \left\{ \frac{\partial P}{\partial t} - \gamma \frac{\partial R}{\partial t} + w \left(\frac{1}{p_0} \frac{\partial p_0}{\partial z} - \gamma \frac{1}{p_0} \frac{\partial p_0}{\partial z} + \frac{1}{M} \frac{\partial M}{\partial z} \right) \right\}$$

As indicated in Part 5 of this section, Q consists of five terms, two of which are of second-order. Of the remaining three, two can be calculated in terms of previously determined quantities:

$$Q_c = -aT'$$

$$Q_k = (k_0 T_0) \frac{\partial^2 T}{\partial z^2} + \left[\frac{3}{2} k_0 \frac{\partial T_0}{\partial z} + \frac{\partial}{\partial z} (k_0 T_0) \right] \frac{\partial T}{\partial z} + \left[\frac{3}{2} \frac{\partial}{\partial z} (k_0 \frac{\partial T_0}{\partial z}) \right] T$$

The heating due to the absorption of insolation can then be calculated from:

$$\bar{T} = \frac{1}{p_0} [Q - Q_c - Q_k]$$

It is anticipated that analysis of the observed winds along the lines indicated in this section will lead to a meaningful estimate of the heating rate, and thus serve as a guide for further theoretical calculations.

SECTION IV

RECOMMENDATIONS FOR FUTURE INVESTIGATIONS

A. Continued Study of the Dynamics of the Thermosphere

If the observed atmospheric motions consisted purely of 3 (or generally N) harmonic components and a prevailing component, then 7 (or generally $2N + 1$) error-free measurements used in conjunction with the straightforward harmonic analysis described in Section II would unambiguously resolve the total wind into its harmonic constituents. In practice, the measurements are not free of error, more than three tidal modes are present, and the overall interval of observation is limited to the roughly 12 hours between evening and morning twilight, or half the period of the diurnal component. In applying straightforward harmonic analysis to the data, it was hoped that these limitations would not preclude the effective resolution of the wind into harmonic components. Unfortunately, the straightforward method does not provide unambiguous results. For this method to work well one would have to increase the number of observations and possibly require that at least one be a daytime observation. The desirability and feasibility of daytime observations are discussed in greater detail in a following paragraph. It is also quite evident that an increased number of observations during the same night would obviate most of the difficulties mentioned above. For the data obtained in the past, ranging from 5 to 7 observations during the same night, a new approach must be sought. The approach to be outlined here applies to the 7-trail series of February 1969. It is anticipated that an extension of this approach will apply to the series involving fewer than 7 observations.

Basically, the approach involves the formation of linear combinations of the seven measurements, each measurement contributing with about equal weight, (i.e., 0, ± 1 , ± 2). By the proper choice of weights, a combination can be formed which is dominated by (but does not exclusively contain) a single periodic component. The variation with height of such a combination will not be completely smooth, owing to the contributions of other components and observational errors. To remove these contaminating contributions, the data must be smoothed by a suitable averaging with respect to height. The "suitable" averaging will depend on the apparent wavelength characteristics of the component dominating the combination, as well as on the apparent wavelength characteristics of the contaminating components. Since all wavelengths in the thermosphere vary with height, the height interval used in the averaging process must also be a function of height. In practice, averaging intervals are determined by a process of trial and error.

This method is currently being applied to the data of February 1969 under Contract No. NAS5-21513. Preliminary results indicate that the amplitude and phase can be obtained as smoothly varying functions of height for the diurnal, semi-diurnal, and ter-diurnal components. The characteristics of other harmonic components revealed by this analysis are more difficult to determine because their amplitudes are small.

The extension of this method to series involving fewer than 7 observations is based on the assumption that, for a particular harmonic component, the variation of its phase with height will be essentially the same as in February 1969. Stated differently, this assumption implies that the phase for a particular harmonic component will be equal to the phase for February 1969 to within a constant which is independent of height.

The importance of being able to resolve the observed winds into identifiable harmonic components is underscored by the formulas of Part 9 of Section III. These formulas allow the calculation of pressure, density, and temperature variations, as well as ion densities and heating rates, from the values of the horizontal winds pertaining to the same mode. In brief, if the horizontal winds can be resolved into identifiable harmonic components, they become the means for measuring quantities which are not easily measured directly.

An estimate of the importance of non-linear effects in a finite height range (80-100 km) was made in Section III. A second-order calculation can be extended outside this range if the viscosity, conductivity, and ion drag effects are included in the calculation. With these effects included, an analytic calculation is no longer possible, but a numerical calculation is. A calculation of this type appropriate to Wallops Island is desirable in order to check the physical concepts embodied in the theoretical calculation against the observations. In particular, the relative importance of the non-linear terms above 100 km remains to be tested.

B. Coordination With Other Measurements

Vapor trails have been deployed from rockets which also carried other measuring systems such as photometers, falling spheres, multiple vapor payloads, and Langmuir probes. All of the multiple payloads resulted in good combined data. A series of Langmuir probes and vapor trails during February 1968 allowed a good comparison of the observations with the theory of redistribution of charge by horizontal winds. Some values of the effective recombination coefficient were obtained and effects of electric fields were evaluated. It is expected that future such observations will furnish additional information. In addition, it is suggested that efforts be made to obtain simultaneous measurements with the increasingly used techniques for measuring ionospheric drifts and with radar meteor wind systems.

C. Measurement of Winds in The Daytime

The requirements for wind measurements during the daytime in order to understand the dynamics of the thermosphere have been discussed often. The importance of diurnal and semi-diurnal tidal components has been established through observation, but these components cannot be correctly evaluated without daytime observations. The expanded use of the ground based methods increases the urgency of the daytime measurements. The

observation with the various RF reflection and scatter technique are very different for the large electron densities during the day than at night and the diurnal variation of meteor in flux causes a reduced altitude range and resolution for the meteor radar wind measurements during the daytime. The application of the vapor trail method for measurement of winds during the daytime has been demonstrated independently by Bedinger (1970) and Best (1970).

D. Proposed Future Investigation

1. Develop technique for resolving observed winds into constituent harmonic components, using the data from the 7 observations of February 1969.
2. Extend and apply this technique to data involving fewer than 7 observations, including available data from other sources.
3. Continue the development of a plane-stratified atmospheric model appropriate to the observations made at Wallops Island. Investigate the importance of non-linear effects above 100 km.
4. Utilize the results of this study in the interpretation and application of data from other sources which are more readily available, but less direct than the vapor trail data, i.e., radio meteor winds and diffusion, ionospheric drifts, airglow variations, vertical and horizontal variations of ionization, sq. currents and electric fields.
5. Propose experimental programs which:
 - (a) aid in calibration or interpretation of other data
 - (b) clarify physical structure of the atmosphere
 - (c) support specific projects or satisfy special requirements.
6. Expand the experimental capability to include daytime usage.